New type of turbulence, or how symmetry results in chaos

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SUMMARY: Pattern formation and transition to chaos in a macroscopic dissipative system is discussed. It is supposed that the system has a continuous family of spatially uniform states, which can be transformed into each other by a certain symmetry transformation. If such a system undergoes instability against spatially periodic perturbations with a *finite* wavenumber, interplay of short-wavelength modes associated with the instability and long-wavelengths modes generated by the symmetry transformation affects the dynamics of the system dramatically. In particular, it may result in *direct* transition from a spatially uniform state to spatiotemporal chaos, analogous to second order phase transition in equilibrium systems. The obtained results may be applied to kinetic of polymerization, to reaction-diffusion problems, to flame propagation, to some hydrodynamic problems, etc.

In the present contribution I would like to call attention of polymer physicists and chemists to a new and very appealing phenomenon recently discovered in dissipative systems, namely to direct transition from a spatially uniform state to spatiotemporal chaos, called *soft-mode turbulence*. The phenomenon is quite common and may be observed, and definitely will be observed, in polymer systems too. I will be happy if I can encourage experts in polymer systems to look for possible manifestations and applications of the soft mode turbulence. Despite detailed study of the phenomenon is associated with sophisticated mathematics, its main features, as usual, can be obtained based upon simple semi-qualitative arguments. In what follows I will focus on this simplified approach to the problem. Those, who are interested in subtle details of the analysis can find them (together with relevant references) in review [1)], which still reflects state of the art.

Allow me firstly remind some basic concepts of the conventional theory of pattern formation in dissipative systems by discussing, e.g., the Rayleigh-Bénard problem (convection in a liquid layer heated from below) as the simplest and most detailed studied example. The convection arises if the temperature difference between the bottom and the top of the layer exceeds a certain threshold: $\Delta T > \Delta T_c$, since the work of buoyancy forces should be big enough to compensate dissipative losses. The important feature of the problem is that it

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possesses *two* different mechanisms of dissipation, namely viscous friction and heat diffusion. Competition between these two mechanisms, where one (viscous losses) prevails in long- and the other (heat diffusion) in short-scale flows, makes a flow with a certain intermediate horizontal scale the most "advantageous", i.e., provides the problem with a certain *intrinsic* horizontal scale d_h. To estimate this scale note, the width of the layer d is the only characteristic spatial scale of the problem. For this reason d_h should be of order d.

Let us make a look at the case from another side. Below the threshold of the convective motion the rest (the, so-called, *conductive*) state of the fluid is spatially uniform in the horizontal plane. Beyond the threshold the convective motion should begin. It is essential, however, that the continuity equation does not allow the convection to be realized in a spatially uniform manner: the bottommost layer can not rise as a whole since there would be no room for the fluid above it to go. In other words, any convective flow must result in loss of spatial uniformity in the horizontal plane. This, in turn, creates a certain spatial scale in this plane, which apparently should be of order d, as it has been already mentioned above.

Such a phenomenon is a particular manifestation of the, so called, *spontaneous symmetry breaking*. In the above mentioned example the symmetry of the spatially uniform state is broken against spatially periodic perturbations with a *finite* wavenumber (*short-wavelengths instability*). The specified prerequisite for the instability (different prevailed mechanisms for stabilization in short- and long-wavelength ranges, or conservation laws and constrains) are pretty common, so the instability may be observed in a wide variety of problems. In addition to the mentioned Rayleigh-Bénard convection, it includes electroconvection in nematics, combustion, frontal polymerization and others, see e.g. [1, 2] for more examples.

Mathematically, the phenomenon is expressed as instability of a spatially uniform state against perturbations containing multipliers of the form

$$\exp(\gamma t + \mathbf{kr})$$

where $\mathbf{k}$ stands for the perturbation wave vector. It is important that the problem has an externally varied quantity (*control parameter*), which controls its stability. If the control parameter is smaller than a certain threshold, the system is stable. Beyond the threshold the instability arises. In Rayleigh-Bénard convection the role of the control parameter may play ΔT, or in a reduced form $\varepsilon \equiv (\Delta T - \Delta T_c) / \Delta T_c$.

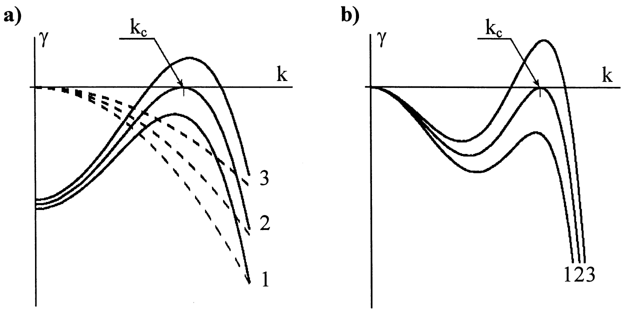

Fig. 1. Growth rate of the instability versus the perturbation wavenumber at three different values of the control parameter (schematically). 1 – below the instability threshold, 2 – exactly at the threshold, 3 – beyond the threshold. The spectrum is supposed to be purely real. (a) Goldstone branch (dash) and unstable modes (solid) correspond to different branches of the spectrum. (b) Goldstone branch and unstable modes correspond to the same branch of the spectrum.

Existence of the characteristic spatial scale in the problem means that close to the threshold unstable perturbations have the value of k from a very narrow band centered around a certain critical wavenumber k_c, see Fig. 1a, solid lines (in the discussed Rayleigh-Bénard problem $k_c \approx 2\pi / d$). If at $t = 0$ the control parameter is set abruptly from a value below the threshold to a value slightly beyond the one, and if the instability arises from small white-noise-like random fluctuations, then only the mentioned narrow band of modes centered about k_c will grow to macroscopic amplitudes. Usually nonlinear coupling of these modes results in stabilization, so finally the system approaches a steady spatially periodic (*patterned*) state with a period close to $2\pi / k_c$ (convective rolls in Rayleigh-Bénard problem, Taylor vortices in Couette flow, etc. [2]).

Suppose now the problem is *degenerate* to the effect that instead of a single, unique spatially uniform state it has a continuous family of these states, which may be transformed into each other by a certain symmetry transformation. Naturally, the class of degenerate problems is narrower than the general class of problems exhibiting pattern formation, but it is still broad enough. First of all it includes patterns arising on traveling fronts, or interfaces. Any position of the front is equivalent but physically different, so the symmetry transforming one spatially uniform state into another is just translation of the front along direction of its propagation. Pattern formation in such a problem in reaction-diffusion systems is discussed in ref. [3] and for traveling solid-vapor interface in laser-sustained evaporation in ref. [4]. Then, it includes the, so called, free-slip convection, see relevant references in refs. [1, 2], and electroconvection in homeotropically aligned nematic layer [1, 5-7]. Next is a generalized Burgers equation [8], and so on.

How does the situation change because of the degeneracy? It is possible to show that the symmetry transformation resulting in the degeneracy generates in the stability spectrum a neutrally stable (*Goldstone*) mode with zero wavenumber [9]. This effect of the symmetry is easy to understand based upon the following argument. Let us consider a steady solution of the governing equation(s), whose stability is examined, and another one, generated by the symmetry transformation, which is very close to the former. The difference between the two solutions may be regarded as a *small perturbation* to the first one, which transforms it into the second. On the other hand, the second solution also satisfies the steady version of the governing equation(s). Because of this fact the perturbation *does not evolve in time*, i.e., it is neutrally stable.

The Goldstone mode often is an origin of a Goldstone branch of the stability spectrum, whose growth rate vanishes at zero wavenumber for *any* value of the control parameter, see Fig. 1. It may be a new branch (Fig. 1a, dash), or the same branch of the spectrum, which is associated with the instability (Fig. 1b). In both the cases we have a new subband of slowly evolving long-wavelength modes, which should be taken into account in study of nonlinear dynamics of the system. The important point is that coupling of long-wavelength modes from the new branch with those centered around k_c destabilizes the system dramatically, so that instead of a steady spatially periodic pattern spatiotemporal chaos may arise just beyond the threshold of the short-wavelength instability.

It should be stressed the chaos arises from a spatially uniform state *directly* in such a manner that the mean amplitudes of turbulent modes gradually increase with increase of the control parameter, while the correlation time gradually decreases. At the threshold of the instability the amplitudes are zero and the correlation time becomes infinitely large. In other words, the chaotic state is *smoothly* matched with the spatially uniform state in the same way as two equilibrium phases are smoothly matched with each other at a second order phase transition point. For this reason the chaos is named *soft-mode turbulence* [5] to emphasize analogy with soft modes in the theory of second order phase transitions.

Another feature of the soft-mode turbulence, which distinguishes it from all other known types of chaos is that the chaos is associated with interplay of different (short and long) spatiotemporal scales, while generation of topological defects (dislocations, etc.) does not play an important role, at least close enough to the threshold of the turbulence.

Experimental evidence of the soft-mode turbulence is obtained in observation of electroconvection in a homeotropically aligned nematic layer [5-7]. In these experiments the layer is sandwiched between two transparent electrodes, while molecules of the nematic are aligned perpendicular to the surfaces of the electrodes. The alignment is imposed due to special treatment of the surfaces, so that anchoring forces rigidly fix axis of molecules contacting with the electrodes. If the applied to the electrodes AC voltage V exceeds a certain threshold V_c, it initiates in the layer convection quite similar to that in Rayleigh-Bénard problem. The convection affects the orientation of the molecules, which in turn affects the refractive index. Thus, in the case of convection instead of an optically homogeneous layer one has a system of focusing and defocusing lenses associated with the convective pattern. It allows to visualize the pattern and to study it with optical methods.

Degeneracy in this problem arises because the electric field "wants" to turn molecules perpendicular to its lines of force, i.e., parallel to the confining surfaces. The alignment imposed by the field conflicts with that imposed by the boundary conditions. Finally, *Fréedericksz* transition occurs and molecules become tilted. In this case they have non-zero projections of their axis on the plane of the layer (*horizontal plane*). On the other hand, since the external factors do not impose any singled out direction in this plane the system is degenerate with respect to arbitrary rotations of all molecules through the same azimuthal angle around vertical axes. Naturally, it is not true close to sidewalls confining the layer's horizontal dimensions ($L_{x,y}$). However, typical values of aspect ratios $L_{x,y}/d$ in the experiments is about 10^3, so the sidewalls play no role in the bulk of the layer.

The important point is that *Fréedericksz* transition *precedes* the convective instability, so the instability arises in a degenerate system. In this situation the azimuthal rotation of the molecules must cause the corresponding rotation of axes of the convective rolls (orientation of convective rolls in the nematic is rigidly connected with orientation of the horizontal projections of the molecules' axes).

A typical snapshot of a pattern, observed under the specified conditions close to the threshold of the electroconvection is presented in Fig. 2. White lines in the snapshot may be attributed to axes of the convective rolls. It is clearly seen from the figure that the system exhibits short-range ordering related to the convective instability.

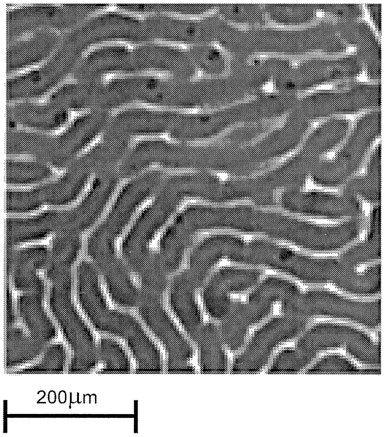

Fig. 2. A snapshot of a convective pattern corresponding to the soft-mode turbulence [6]. Electroconvection in a homeotropically aligned nematic layer. The layer thickness is 50μm, AC frequency f= 100Hz, $\varepsilon \equiv (V^2 - V_c^2)/V_c^2 = 0.10$.

However, at long range the ordering is destroyed because of random long-wavelength modulations originated in the Goldstone branch of the spectrum. The pattern is dynamical. It slowly evolves in time but never reaches any steady state.

Thus, the soft-mode turbulence is a solid fact, which has the theoretical explanation and the experimental evidence. In addition to new light, which it sheds on our understanding of such fascinating phenomena as chaos and complexity, it may have important practical applications, say for turbulization of reagents in chemical reaction, or turbulization of flames in slow combustion regimes. The conditions required for manifestation of the soft-mode turbulence are quite common and I appeal to experts in different fields to pay attention to this new phenomenon.

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